fifth order in ε in /10/ is also found to be in accord with (6.5).

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LINEAR WAVES IN A FLUID FLOW WITH CONSTANT VORTICITY LOCATED UNDER AN ICE BLANKET*

A.V. MARCHENKO and I.V. PROKHOROV

The linear dynamics of periodic waves on the surface of a fluid layer of finite depth located under an ice blanket which is simulated by an elastic plate is considered. The fluid particles in the unperturbed state move at a constant horizontal velocity, the profile of which has a linear shift along the vertical. It is shown that several type of waves exist which propagate at the same frequency. The number of waves depends on the frequency, the flow parameters in the fluid and the physico-mechanical parameters of the ice blanket. The problem of the diffraction of waves of fixed frequency on the edge of a semi-infinite elastic plate which floats on the surface of the fluid is considered. The problem is reduced to the solution of Laplace's equation in the strip with specified asymptotic forms at infinity and with boundary conditions on the sides of the strip which have a discontinuity at a point corresponding to the edge of the ice and contact-boundary conditions on the edge of the plate. The solution is constructed using the Wiener-Hopf method. The reflection and transmission coefficients of the waves across the edge of the plate are determined. The results obtained are analysed using the actual parameters of sea ice.

In investigations of the dynamics of waves in a fluid layer with a constant vorticity

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with a free surface, it has been shown /1, 2/ that the equations of motion of the fluid have an integral which is an analogue of the Cauchy-Lagrange integral. The problem of the diffraction of surface and hydro-acoustic waves at the edge and at the inhomogeneities of an elastic plate floating on the surface of an infinitely deep fluid /3-6/ and the problem of the diffraction of internal waves in a layer of an exponentially stratified fluid on the edge of an ice field and on the edge of a semi-infinite film which changes the surface tension of the fluid /7-9/ are considered.

1. Planar motions in a layer of a heavy fluid with an unperturbed depth H are considered. The origin of the rectangular system of Cartesian coordinate x, y is located on the unperturbed surface of the fluid and the y-axis is directed vertically downwards.

The vortex vector has a magnitude $\Omega = \partial u/\partial y - \partial v/\partial x$ (v, u are the vertical and horizontal components of the velocity of the fluid particles). It follows from the equation for the conservation of vorticity $d\Omega/dt = 0$ that if, at the initial instant of time, the velocity of the fluid particles has the form

$$u = \Omega y + V + u',\ v = v',\ \Omega = (V_d - V)/H = \text{const}$$

where V and V_d are the velocities of the unperturbed fluid flow on its surface and at the bottom, then, the vorticity does not change at later instants of time. Hence, in the case of the velocities u' and v', a potential φ exists:

$$\Delta\varphi = 0,\ (u', v') = \nabla\varphi \tag{1.1}$$

The equations of motion of the fluid have the integral /1/

$$\frac{\partial\varphi}{\partial t} + (\Omega y + V)\frac{\partial\varphi}{\partial x} + \frac{1}{2}\left[\left(\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial y}\right)^2\right] - \Omega\psi + \frac{P}{\rho} - gy = f(t) \tag{1.2}$$

(P and ρ are the pressure and density of the fluid and ψ is the stream function).

It is assumed that a thin elastic plate, which simulates an ice blanket, floats on the surface of the fluid. The pressure under the plate P_i is then associated with the atmospheric pressure $P_a = \text{const}$ by the relationship /2-4/

$$\frac{P_i - P_a}{\rho} = -D\frac{\partial^4}{\partial x^4}\eta, \quad D = \frac{Eh^3}{12(1-\nu^2)\rho} \tag{1.3}$$

Here, E and ν are Young's modulus and Poisson's ratio of ice, h is the thickness of the ice and η is the magnitude of the flexure of the plate above the horizontal equilibrium position.

If it is assumed that there are no fluid-free cavities formed under the ice blanket, the magnitude of the flexure is related to the velocity of the fluid particles on the surface by the kinematic boundary condition

$$L\eta = \frac{\partial\varphi}{\partial y}, \quad y = \eta \tag{1.4}$$
$$L = \partial/\partial t + (\Omega\eta + V\partial\varphi/\partial x)\partial/\partial x$$

A no-flow condition is satisfied on the bottom

$$\partial\varphi/\partial y = 0,\ y = H \tag{1.5}$$

By considering small-amplitude motions, we obtain a linear boundary-value problem for the Laplace Eq.(1.1) in the domain $y \in (0, H)$, $x \in (-\infty, \infty)$ with boundary conditions (1.5) and the condition

$$L_t^2\varphi - \Omega L_t\psi - g\frac{\partial\varphi}{\partial y} - D\frac{\partial^4}{\partial x^4}\frac{\partial\varphi}{\partial y} = 0, \quad y = 0 \tag{1.6}$$
$$L_t = \frac{\partial}{\partial t} + V\frac{\partial}{\partial x}$$

which follows from (1.2)-(1.4).

Let us write the solution, which is of the type of periodic plane waves, in the form

$$\varphi = A\,\text{ch}\,[k(y-H)]\exp[i(kx + \omega t)] \tag{1.7}$$

This potential identically satisfies Laplace's equation and the boundary condition (1.5). By substituting expression (1.7) into the boundary condition (1.6), we find that ω is a root of the dispersion equation

$$G(k,\omega) \equiv (\omega - \omega_+)(\omega - \omega_-) = 0, \quad \omega_\pm = -Vk - {}^1/_2\Omega \,\mathrm{th}\, kH \left(1 \pm \sqrt{1 - 4k(g + Dk^4)\Omega^{-2}\,\mathrm{cth}\, kH}\right) \tag{1.8}$$

This dispersion relationship has two branches in the ω, k plane, a positive and a negative branch, which are defined by the equations $\omega = \omega_\pm(k)$ which are invariant under the substitutions $\omega \to -\omega$, $k \to -k$. Both of these branches of the dispersion curve are therefore antisymmetric about the origin of coordinates.

For small k, we find from (1.8) that

$$c_\pm = k^{-1}\omega_\pm = -V - {}^1/_2\Omega H\left(1 \pm \sqrt{1 - 4\lambda}\right), \quad \lambda = g/(H\Omega^2) \tag{1.9}$$

Let us put

$$V_d = 0, \quad 0 \leqslant V \lesssim 10\,\mathrm{cm/s}, \quad 10\ \mathrm{m} \lesssim H \lesssim 10^3\ \mathrm{m} \tag{1.10}$$

The estimates (1.10) are related to the real velocity scales of the flows and the depths in the ocean.

It follows from (1.10) that $\lambda \gtrsim 10^4$. The velocities $c_\pm$ of the long waves therefore have different signs. The positive branch of the dispersion relationship is located in the first and fourth quadrants and the negative branch is located in the second and third quadrants of the ω, k plane when k is small.

Let us consider the case when $D = 0$. Typical plots of the curves $\omega = \omega_\pm(k)$ are shown in Fig.1. It follows from (1.8) that $\omega_\pm \to -\infty$ when $k \to +\infty$ and $\omega_\pm \to +\infty$ when $k \to -\infty$. On the dispersion curves ω_+ when $k > 0$ and ω_- when $k < 0$, there is a local maximum and minimum respectively to which the frequencies ω_* and $-\omega_*$ correspond. When $\omega = 0$, Eq.(1.8) has a non-zero solution $k = k_1^\pm$ (Fig.1) which corresponds to waves which are blocked by the flow and have zero phase velocity. When $|\omega| < \omega_*$, there are four waves with a frequency ω and different wave numbers $k_1^1, \ldots, k_1^4$ (Fig.1). The waves $k_1^{1,2}$ propagate along the fluid flow while the waves $k_1^{3,4}$ propagate against the flow. When $|\omega| > \omega_*$, there are two waves with a frequency ω and different wave numbers which propagate along the flow (Fig.1). By putting $V = 10$ cm/s and $H = 100$ m, we obtain the estimates:

$$\omega_* \approx 20\ \mathrm{s}^{-1}, \quad 10^3\ \mathrm{m}^{-1} \lesssim |k_1^1| \lesssim 1.5\cdot 10^3\ \mathrm{m}^{-1},$$
$$|k_1^2| \lesssim 60\ \mathrm{m}^{-1}, \quad |k_1^3| \lesssim 300\ \mathrm{m}^{-1}, \quad 300\ \mathrm{m}^{-1} \lesssim |k_1^4| \lesssim 10^3\ \mathrm{m}^{-1},$$
$$0 < \omega < \omega_* \tag{1.11}$$

It is seen that waves from the capillary range are necessarily present among the waves with a frequency $|\omega| < \omega_*$.

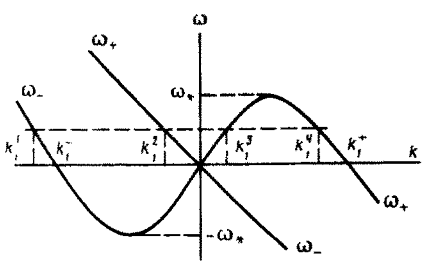

Fig.1

Let us consider the case when $D > 0$. It is found from (1.8) that $\omega_+ \to +\infty$ when $k \to \pm\infty$ and $\omega_- \to -\infty$ when $k \to \pm\infty$. Plots of the dispersion curves $\omega = \omega_+(k)$ and $\omega = \omega_-(k)$ for different values of D have a similar form in the domains $k < 0$ and $k > 0$ respectively (they are represented by the curves $\omega_\pm$ in Fig.2).

When $D < D_1$, where D_1 is the solution of the system of equations

$$\partial\omega_+/\partial k = \partial^2\omega_+/\partial k^2 = 0$$

there is a local maximum and a local minimum on the branches ω_+ when $k > 0$ and ω_- when $k < 0$ to which the frequencies $\pm\omega^{m,s}_{*,1}$ $\pm\omega^{m,s}_{*,2}$ correspond (Fig.2). When $D < D_2$, where D_2 is the solution of the equations

$$\partial\omega_+/\partial k = \omega_+ = 0$$

the condition $\omega^m_{*,1} < 0$ is satisfied.

Typical plots of ω_+ when $k < 0$ and ω_- when $k > 0$ are represented in Fig.2 by the curves $\omega^m_\pm$.

In this case, Eq.(1.8), when $\omega = 0$, has four non-zero roots $k^m_{1,\pm}$, $k^m_{2,\pm}$ (Fig.2) which correspond to waves which are blocked by the flow and have a zero phase velocity. Let us use the notation

$$\omega_{\min} = \min(\omega^m_{*,2}, -\omega^m_{*,1}), \quad \omega_{\max} = \max(\omega^m_{*,2}, -\omega^m_{*,1})$$

When $|\omega| < \omega_{\min}$, Eq.(1.8) has six roots which correspond to six waves with a frequency ω, three of which propagate along the flow and three against the flow. When $\omega_{\min} < |\omega| < \omega_{\max}$, Eq.(1.8) has four roots which corresponds to four waves with a frequency ω, one of which, when $\omega_{\min} = \omega^m_{*,2}$, propagates against the flow while three propagate with the flow and, when $\omega_{\min} = -\omega^m_{*,1}$, one wave propagates along the flow and three waves propagate against the flow. When $|\omega| > \omega_{\max}$, Eq.(1.8) has two roots which corresponds to two waves with a frequency ω, which travel in opposite directions.

When $D_2 < D < D_1$, the condition $\omega^s_{*,2} > \omega^s_{*,1} > 0$ is satisfied. Typical plots of ω_+ when $k > 0$ and ω_- when $k < 0$ are represented in Fig.2 by the curves $\omega^s_\pm$. In this case, Eq.(1.8) does not have any solution differing from zero when $\omega = 0$. When $|\omega| < \omega^s_{*,1}$, Eq.(1.8) has two roots corresponding to two waves with a frequency ω which travel in opposite directions. When $\omega^s_{*,1} < |\omega| < \omega^s_{*,2}$, Eq.(1.8) has four roots corresponding to four waves with a frequency ω. One of these propagates along the flow and three propagate against it. When $|\omega| > \omega^s_{*,2}$, Eq.(1.8) has two roots corresponding to two waves with a frequency ω, which travel in opposite directions.

When $D > D_1$, Eq.(1.8) has two roots $k_2^{1,2}$ which correspond to two waves with a frequency ω which travel in opposite directions. Typical plots of ω_+ when $k > 0$ and ω_- when $k < 0$ are represented in Fig.2 by the curves $\omega^l_\pm$.

Let us now consider a case which is of interest from the point of view of physical applications. We let a homogeneous ice plate float on the surface of a fluid and, following /10/, we put

$$E = 10^9 \text{ N/m}^2, \ \rho = 900 \text{kg/m}^3, \ h = 1 \text{ m} \tag{1.12}$$

It follows from (1.12) that the dispersion curves correspond to the case when $D > D_1$.

2. Let us consider the problem of the diffraction of periodic waves of fixed frequency ω at the edge of an ice blanket floating on the surface of a fluid in the domain $x > 0$. All of the functions occurring in the problem depend on the time via a factor $e^{i\omega t}$ which is subsequently omitted. It is assumed that the parameters of the ice and the fluid satisfy the estimates (1.11) and (1.12).

A source of periodic perturbations is located either at $\pm\infty$ along x or on the edge of the ice at the point $x = y = 0$. It is therefore necessary to specify the amplitudes of the waves which arrive on the edge of the ice from infinity as well as the force and moment acting on the edge

$$D\partial^2\eta/\partial x^2 = M, \quad D\partial^3\eta/\partial x^3 = N, \quad x \to +0 \tag{2.1}$$

Here, M and N are the amplitude of the moment and the force.

We note that the flow rate of the fluid across any closed volume located within the domain of the motion is equal to zero. However, the flow rate of the fluid close to the edge of the plate may be non-zero. The contact-boundary condition

$$V(\eta^- - \eta^+) = Q, \quad \eta^\pm = \lim_{x\to\pm 0} \eta \tag{2.2}$$

must therefore be satisfied on the edge.

The quantity Q must be determined from the solution of an internal problem on the flow round the edge of the plate. When $V = D = 0$, conditions (2.1) and (2.2) are satisfied identically.

When $x \to \pm\infty$, the solution of the problem must have the asymptotic forms

$$\varphi \to \varphi_t^- + \varphi_r^-, \ x \to -\infty; \quad \varphi \to \varphi_t^+ + \varphi_r^+, \ x \to +\infty \tag{2.3}$$
$$\varphi_t^- = T_1^-\theta(k_1^1) + T_2^-\theta(k_1^2), \quad \varphi_t^+ = T_2^+\theta(k_2^2)$$
$$\varphi_r^- = R_1^-\theta(k_1^3) + R_2^-\theta(k_1^4), \quad \varphi_r^+ = R_2^+\theta(k_2^1)$$
$$\theta(k) = [\operatorname{ch} k(y - H)/\operatorname{ch} kH]e^{ikx}$$

The potentials φ_l^- and φ_r^- correspond to waves travelling to the right and φ_r^+ and φ_l^+ correspond to waves travelling to the left.

Hence, in order to solve the problem under consideration, it is necessary to find functions φ and ψ which are harmonic in the domain $y \in (0, H)$, $x \in (-\infty, \infty)$ and which satisfy boundary conditions (1.5) and (1.6), where it is necessary to put $D = 0$ when $x < 0$ and $D > 0$ when $x > 0$. The solution of this problem, as will be seen from what follows, is not unique and depends on six arbitray constants $A_0, \ldots, A_5$ which are determined from the contact-boundary conditions (2.1), (2.2) and from the specification of the asymptotic forms of the solutions at infinity (2.3).

Let us represent the solution of the problem in the form of a Fourier integral

$$\varphi = \frac{1}{2\pi i} \int_{-\infty}^{\infty} p(k)\,\theta(k)\,dk \tag{2.4}$$

$$\psi = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{k} p(k) \frac{\partial}{\partial y} \theta(k)\,dk$$

It satisfies Laplaces equation and the boundary condition on the bottom (1.5) identically.

Instead of (1.6), let us consider the following boundary conditions for the functions φ and ψ when $y = 0$:

$$L_l^2\varphi - \Omega L_l\psi - g\frac{\partial\varphi}{\partial y} + \mu\left(\alpha_1\frac{\partial\varphi}{\partial x} + i\alpha_2\frac{\partial\varphi}{\partial y} + i\alpha_3\frac{\partial^2\varphi}{\partial x^2} + \alpha_4\frac{\partial^2\varphi}{\partial x\,\partial y}\right) = 0, \quad x < 0 \tag{2.5}$$

$$L_l^2\varphi - \Omega L_l\psi - g\frac{\partial\varphi}{\partial y} - D\frac{\partial^4}{\partial x^4}\frac{\partial\varphi}{\partial y} + \mu\frac{\partial\varphi}{\partial x} = 0, \quad x > 0$$

The coefficients $\mu, \alpha_1, \ldots, \alpha_4$ are chosen such that the roots of the equation $G_1 = 0$ are displaced into the lower half plane while the roots of the equation $G_2 = 0$ are displaced into the upper half plane of the complex variable k. This enables us to carry out factorization in the Wiener-Hopf method and to take account of all types of non-decaying waves of freqeuncy ω in the solution of the problem which is obtained by passing to the limit when $\mu \to 0$.

Let us rewrite the boundary conditions (2.5), taking account of (2.4), in the form /6/

$$\int_{L_1} pG_1 e^{ikx}\,dk = 0, \quad x < 0; \quad \int_{L_2} pG_2 e^{ikx}\,dk = 0, \quad x > 0 \tag{2.6}$$

$$G_1 = G(D = 0) + i\mu G_-, \quad G_2 = G(D > 0) + i\mu k$$

$$G_- = k(\alpha_1 - \alpha_3 k - \operatorname{th} kH(\alpha_2 + \alpha_4 k))$$

$$\operatorname{Im} k < 0, \ k \in L_1, \ |k| \to \infty; \ \operatorname{Im} k > 0, \ k \in L_2, \ |k| \to \infty$$

Assuming that the coefficient μ can take values which may be as small in modulus as desired, we find that the equations $G_{1,2} = 0$ have roots which differ from $k_1^1, \ldots, k_1^4, k_2^1, k_2^2$ by small imaginary additions

$$\Delta k_1^j \approx -\frac{i\mu G_-(k_1^j)}{\partial G/\partial k\,(D = 0, k_1^j)} \tag{2.7}$$

$$\Delta k_2^{1,2} \approx -\frac{i\mu k_2^{1,2}}{\partial G/\partial k\,(D > 0, k_2^{1,2})}$$

From the condition $\operatorname{Im}\Delta k_2^{1,2} > 0$, we find that $\mu > 0$. The coefficients α_j can be chosen so that the inequality $\operatorname{Im}\Delta k_1^j < 0$ is satisfied.

Conditions (2.6) will be satisfied if we put

$$pG_1 = \Phi^-, \ pG_2 = \Phi^+ \tag{2.8}$$

where Φ^+ and Φ^- are functions which are analytic in the upper and lower half planes of k respectively.

Let us now represent $G_{1,2}$ in the form

$$G_1 = g_1(V^2 - i\mu(\alpha_3 + \alpha_4))\Pi^+ / (k^2 + \mu^2) \tag{2.9}$$

$$G_2 = g_2 D(k^2 + \mu^2)^{5/2}\Pi^-$$

$$\Pi^+ = \prod_{j=1}^{4}(k - k_1^j), \quad \Pi^- = (k - k_2^1)(k - k_2^2)$$

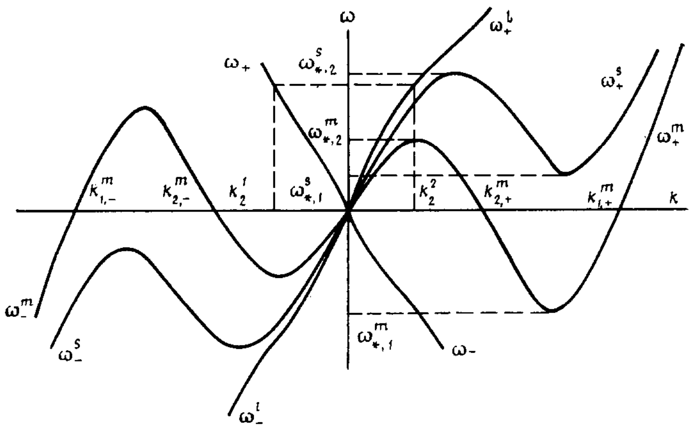

Fig.2

The functions $g_{1,2}$ are analytic, they do not have zeros and tend to unity when $|k| \to \infty$ close to the real axis of k and can be factorized

$$g_j = g_j^+ g_j^-, \quad g_j^{\pm} = \exp\left[\pm \frac{1}{2\pi i} \int\limits_{-\infty \mp i0}^{\infty \mp i0} \frac{\ln g_j}{k-t}\, dk\right]$$

The functions g_j^+ and g_j^- are analytic, they do not have any zeros and tend to unity when $|k| \to \infty$ in the upper and lower half planes respectively.

From (2.8) and (2.9), we find

$$\frac{\Phi^+ \Pi^+ g_1^+}{D g_2^+ (k + i\mu)^{5/2}} = \frac{\Phi^- \Pi^- (k - i\mu)^{5/2} g_2^-}{V^2 g_1^-} = F \tag{2.10}$$

The function F is analytic in the whole k plane. It follows from the convergence of the integral in (2.4) that $p(k) = O(|k|^{-1-\varepsilon})$, $\varepsilon > 0$, $|k| \to \infty$. Using formulae (2.8) and (2.9), we get

$$F = O(|k|^{11/2-\varepsilon}), \quad |k| \to \infty$$

According to Liouville's theorem, we find from this that F is a fifth-degree polynomial

$$F(k) = \sum_{j=0}^{5} A_j k^j \tag{2.11}$$

The constants A_j are determined from (2.1)-(2.3). By substituting series (2.11) into (2.10), we find Φ^+ and Φ^-. We then determine p using formulae (2.8). On substituting p into (2.4), evaluating the integrals of the residues and putting $\mu = 0$ in the solution, we find that

$$\varphi = \varphi_p^- + \varphi_f^-, \ x < 0; \quad \varphi = \varphi_p^+ + \varphi_f^+, \ x > 0 \tag{2.12}$$

$$\varphi_p^- = \sum_{l=1}^{4} F(k_1^l)\chi_l^- \theta(k_1^l), \quad \varphi_f^- = \sum_{j=-1}^{-\infty} F(b_1^j)\chi_j^- \theta(b_1^j)$$

$$\varphi_p^+ = \sum_{l=1}^{2} F(k_2^l)\chi_l^+ \theta(k_2^l), \quad \varphi_f^+ = \sum_{j=1}^{\infty} F(b_2^j)\chi_j^+ \theta(b_2^j)$$

$$\chi_l^- = \left.\frac{V^2 g_1^-}{\Pi^- g_2^- \partial G_1/\partial k}\right|_{k=k_1^l} (k_1^l)^{-5/2} e^{i\varphi_l^-} \tag{2.13}$$

$$\varphi_{1,2}^- = \frac{\pi}{2}, \quad \varphi_{3,4}^- = 0$$

$$\chi_l^+ = \left.\frac{D g_2^+}{\Pi^+ g_1^+ \partial G_2/\partial k}\right|_{k=k_2^l} (k_2^l)^{5/2} e^{i\varphi_l^+}$$

$$\varphi_1^+ = \frac{\pi}{2}, \quad \varphi_2^+ = 0$$

In formulae (2.13) and subsequently it is necessary to put $k_n^j = b_n$, $\varphi_j^\pm = 0$ and replace l by j. The quantities b_n^j $(n = 1, 2, j \in Z)$ are the complex roots of the equations $G_n = 0$ lying in the upper and lower half planes of k when $j > 0$ and $j < 0$ respectively. When extracting a root from b_n^j it is necessary to assume that the k plane has been cut along the positive part of the imaginary axis when calculating χ_j^- and along the negative part of the imaginary axis when calculating χ_j^+. For large $|j|$, the roots b_l^j have the asymptotic forms

$$b_1^j = i\frac{\pi}{H}\left(j + \frac{1}{2}\right) - i\frac{H}{\pi}\frac{\Omega V + g}{jV^2} + O(j^{-2})$$

$$b_2^j = i\frac{\pi}{H}j + O(j^{-3})$$

From the linearlization condition (1.4), we find

$$\eta = \sum_{l=1}^{2} F(k_2^l)\Phi_l^+ \exp(ik_2^l x) + \sum_{j=1}^{\infty} F(b_2^j)\Phi_j^+ \exp(ib_2^j x), \quad x > 0 \tag{2.14}$$

$$\eta = \sum_{l=1}^{4} F(k_1^l)\Phi_l^- \exp(ik_1^l x) + \sum_{j=-1}^{-\infty} F(b_1^j)\Phi_j^- \exp(ib_1^j x), \quad x < 0$$

$$\Phi_l^\pm = i\frac{k_{2,1}^l \chi_l^\pm \operatorname{th}(k_{2,1}^l H)}{\omega + Vk_{2,1}^l}$$

From (2.1)-(2.3), we obtain a system of six linear algebraic equations for determining the unknown coefficients A_j of the polynomial F:

$$F(k_1^{1,2})\chi_{1,2}^- = T_{1,2}^-, \quad F(k_2^2)\chi_2^+ = T_2^+ \tag{2.15}$$

$$V\left(\sum_{l=1}^{4} F(k_1^l)\Phi_l^- - \sum_{l=1}^{2} F(k_2^l)\Phi_l^+ + \sum_{j=-1}^{-\infty} F(b_1^j)\Phi_j^- - \sum_{j=1}^{\infty} F(b_2^j)\Phi_j^+\right) = Q$$

$$\sum_{l=1}^{2} (k_2^l)^2 F(k_2^l)\Phi_l^+ + \sum_{j=1}^{\infty} (b_2^j)^2 F(b_2^j)\Phi_j^+ = -MD^{-1}$$

$$\sum_{l=1}^{2} (k_2^l)^3 F(k_2^l)\Phi_l^+ + \sum_{j=1}^{\infty} (b_2^j)^3 F(b_2^j)\Phi_j^+ = iND^{-1}$$

The reflection coefficients $R_{1,2}^-$ and R_2^+ are determined from relationships (2.3) using (2.12)

$$R_{1,2}^- = F(k_1^{3,4})\chi_{3,4}^-, \quad R_2^+ = F(k_2^1)\chi_1^+$$

The velocity potential φ, defined by formulae (2.12), is continuous in the whole of the domain of motion, including the point $x = y = 0$ and has derivatives of any order everywhere apart from the point $x = y = 0$. The derivatives $\partial\varphi/\partial x$, $\partial^4\eta/\partial x^4$ when $x > 0$ and $\partial\varphi/\partial y\, \partial\eta/\partial x$ when $x < 0$ have singularities at the point $x = y = 0$ and increase on approaching the edge in inverse proportion to the distance from it. The increase in the velocities $\partial\varphi/\partial x$ and $\partial\varphi/\partial y$ is explained by the fact that the flow rate of the fluid across a circle of small radius with its centre at the origin of coordinates is equal to zero. The derivatives $\partial\varphi/\partial x$ when $x < 0$ and $\partial\varphi/\partial y$, $\partial^n\eta/\partial x^n$ $(n \leqslant 3)$ when $x > 0$ are finite as the edge is approached but the elevation of the surface of the fluid η has a discontinuity of the first kind at the point $x = y = 0$.

The functions $\varphi_p^\pm$ determine the asymptotic forms of φ when $x \to \pm\infty$ and correspond to non-decaying periodic waves which bring energy to and carry away energy from the edge of the ice to infinity. When $H \to \infty$, $y \to H$, the functions $\varphi_p^\pm$ decay exponentially. The terms $\varphi_f^\pm$ make a contribution to the solution close to the edge and decay exponentially far from this edge. When $H \to \infty$, $y \to H$, the exponential decay of $\varphi_f^\pm$ has an oscillatory form.

3. Let us now draw some conclusions. The occurrence in a fluid with a non-zero velocity V on the surface leads to a state of affairs where, at the point $x = y = 0$, it is necessary to set out a contact-boundary condition (2.2) which specifies the flow rate of the fluid Q at a given point. Condition (2.2) corresponds to the splashing of the wave across the edge of the ice or to a periodic fluid source or sink arranged around the edge. If $V = 0$, then $Q = 0$ and condition (2.2) is satisfied identically.

It follows from (1.11) and (1.12) that there is a critical frequency ω_* such that, when $\omega > \omega_*$, the wave numbers of the reflected waves $k_1^{3,4}$ become complex. Hence, when $\omega > \omega_*$, a reflected perturbation decays asymptotically as it becomes more remote from the edge of the ice in the domain $x < 0$.

The diffraction of the surface wave k_1^2 (Fig.1) with a frequency $\omega < \omega_*$ on the edge is the cause of the generation of the wave k_1^4 from the capillary range which propagates towards the pure water. If $V = 0$, that is, there is no flow on the surface of the fluid and no capillary waves are excited.

A short wave k_1^1 arriving at the edge as a result of diffraction excites long waves under the ice and on the pure water.

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